Robust Nonlinear Integral Control by Partial-State and Output Feedback*

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Abstract. It is well-known from linear systems theory that an integral control law is needed for asymptotic set-point regulation under parameter perturbations. This paper presents a similar result for a class of nonlinear systems in the presence of an unknown equilibrium due to uncertain nonlinearities and dynamic uncertainties. Both partial-state and output feedback cases are considered. A procedure for robust nonlinear integral controller design is presented and illustrated via a practical example of fan speed control.

1 Introduction

It is widely recognized that an integral controller is inherently robust in the face of model and controller parameter variations. The value of integral control in achieving robust asymptotic regulation has recently been exploited for nonlinear uncertain systems – see, e.g., [1, 2, 3, 7, 8] and references therein. In [1], Freeman and Kokotović propose a backstepping scheme for robust integral control of a class of nonlinear systems with unknown nonlinearities. Global set-point regulators with disturbance rejection property are constructed at the price of assuming full-state information and the relative degree being equal to the system order. Both assumptions in [1] are relaxed by Khalil [8] by means of his "high-gain observers" techniques complemented by the idea of saturating the controller outside a compact set of interest. As a consequence of the "worst-case" design, the results in [8] are of regional and semiglobal types.

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The purpose of this paper is to propose global regulation results for a class of nonlinear systems with disturbances combining those in [1, 2, 8], i.e., we do consider unmeasured zero-dynamics and uncertain nonlinearities. Both partial-state and output feedback control cases will be investigated. The proposed results are a complement of our previous paper [5, 4] that are based on nonlinear ISS small-gain theorems [6]. Our results in this paper can also be considered as an extension of earlier adaptive control work [9, 10] when the underlying disturbances are simply unknown and constant parameters. As in the work of Khalil [8] (also see [5, 4, 6]), we characterize the system of dynamic uncertainties via Sontag's ISS ("input-to-state stability") and ISS-Lyapunov functions [11]. It is shown in this paper that our robust integral controllers reduce to linear control laws under some sufficient conditions one of which is that input-output gain associated with the dynamic uncertainties is linearly bounded. Moreover, our controller becomes a classical PI control law if, additionally, the system is of relative degree one, or the uncertainty falls into the input space (i.e., strict matching condition).

Section 2 formulates the control problem of interest. Section 3 deals with robust integral control by partial-state feedback and describes the control design scheme. Section 4 states and proves our main theorems with an extension to output feedback briefly discussed in Section 5. An illustrative example of fan speed control is provided in Section 5. Finally, Section 6 closes the paper with some concluding remarks.

2 Problem statement

We address the problem of robust global integral control for a class of single-input single-output (SISO) nonlinear systems of the form

$$
\begin{array}{rcl}
\dot{\eta} & = & q(\eta, y) \\
\dot{\xi}_1 & = & \xi_2 \ + \ \varphi_1(\xi_1)\theta(\eta, y) \\
\dot{\xi}_2 & = & \xi_3 \ + \ \varphi_2(\xi_1, \xi_2)\theta(\eta, y) \\
 & \vdots & \\
\dot{\xi}_n & = & u \ + \ \varphi_n(\xi_1, \dots, \xi_n)\theta(\eta, y) \\
y & = & \xi_1
\end{array}
\tag{1}
$$

where $u \in \mathbb{R}$ is the input, $\xi = (\xi_1, \dots, \xi_n) \in \mathbb{R}^n$ is the measured portion of the system state, $\eta \in \mathbb{R}^{n_0}$ is the remaining *unmeasured* state which we refer to as *dynamic uncertainty.* For each $1 \leq i \leq n$, φ_i is a known and sufficiently smooth function, and θ is an uncertain nonlinear function. It is of interest to note that, when θ is a vector of unknown constant parameters, (1) is a system in the so-called *block strict-feedback form* [9]. The following assumptions are imposed throughout the paper.

(A1) For any pair of (η_r, y_r), there are two (unknown) constants p_1 and p_2 such that

$$|\theta(\eta, y) - \theta(\eta_r, y_r)| \leq p_1 \kappa_1(|\eta - \eta_r|) + p_2 \kappa_2(|y - y_r|)$$

with κ_1 and κ_2 smooth functions vanishing at the origin and independent of (η_r, y_r).

(A2) For any $y_r \in \mathbb{R}$, there exists a unique η_r in $\mathbb{R}^{n_0}$ such that $q(\eta_r, y_r) = 0$.

(A3) Letting $x_1 = y - y_r$ and $z = \eta - \eta_r$, the derived system

$$\dot{z} \ = \ q(z + \eta_r, x_1 + y_r) \ := \ q_0(z, x_1) \tag{2}$$

has an ISS-Lyapunov function $V_0(z)$ (see [11]). Namely, V_0 is a positive-definite and proper function that satisfies

$$\frac{\partial V_0}{\partial z}(z) q_0(z, x_1) \ \leq \ -\alpha_0(|z|) \ + \ \gamma_0(|x_1|) \tag{3}$$

where α_0 and γ_0 are class-$\mathcal{K}_\infty$ functions.

Given a reference set-point y_r, the control objective considered in the paper is to find a dynamic, partial-state feedback law of the form

$$\dot{\chi} \ = \ \nu(\chi, \xi) \ , \quad u \ = \ \mu(\chi, \xi) \tag{4}$$

such that the following properties hold *globally* for the closed-loop system (1)-(4):

(P1) The solutions $(\eta(t), \xi(t), \chi(t))$ are well-defined and are bounded over $[0, \infty)$.

(P2) The tracking error $y(t) - y_r \to 0$ as $t \to \infty$.

In Section 5, a dynamic output-feedback solution will be provided using output measurements y.

This issue of set-point regulation in the presence of nonvanishing disturbances has been studied by Freeman and Kokotović [1, 2] for systems without zero-dynamics assuming full-state information. More recently, Khalil [8] looked at the problem of output feedback integral control for systems with unmeasured zero-dynamics. Using his "high-gain observers" techniques, Khalil [8] has proposed interesting regional and semiglobal results and has brought to light connections between his results and traditional PI control. Here, we are interested in global results, relying upon either partial-state or output information.

3 Robust integral control by partial-state feedback

The main purpose of this section is to develop a constructive procedure for robust nonlinear integral controller design. We start with the simplest case when the relative degree of system (1) is one, i.e., $n = 1$. Then, we study the case when the relative degree is larger than one, i.e., $n > 1$. The stability analysis is carried out in Section 4 by selecting appropriate design functions and under small-gain type (local) conditions.

3.1 The relative-degree-one case: $n = 1$

In this case, (1) becomes

$$
\begin{array}{rcl}
\dot{\eta} & = & q(\eta, y) \\
\dot{\xi}_1 & = & u \ + \ \varphi_1(\xi_1)\theta(\eta, y) \\
y & = & \xi_1
\end{array}
\tag{5}
$$

Let $\theta^\star := \theta(\eta_r, y_r)$ and $p^\star := \max\{p_1, p_2, p_1^2, p_2^2\}$ denote unknown and constant parameters and let $\widehat{\theta}$ and $\widehat{p}$ denote their respective estimate. Consider the function

$$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2}(\widehat{\theta} - \theta^\star)^T \Gamma^{-1} (\widehat{\theta} - \theta^\star) + \frac{1}{2\lambda}(\widehat{p} - p^\star)^2$$

where $x_1 = y - y_r$ is the tracking error, and $\Gamma = \Gamma^T > 0$ and $\lambda > 0$ are adaptation gains.

Taking the differentiation of V_1 with respect to time yields

$$\begin{aligned}\dot V_1 &\le x_1\left(u+\varphi_1(\xi_1)\widehat\theta+\widehat p x_1\widehat\varphi_1\right)+\kappa_1^2(|z|)\\ &\quad +(\widehat\theta-\theta^\star)^T\Gamma^{-1}\left(\dot{\widehat\theta}-\Gamma x_1\varphi_1\right)\\ &\quad +\lambda^{-1}(\widehat p-p^\star)\left(\dot{\widehat p}-\lambda x_1^2\widehat\varphi_1\right) \qquad (6)\end{aligned}$$

where $\widehat\varphi_1$ is a nonnegative smooth function such that

$$x_1\varphi_1(\theta(\eta,y)-\theta(\eta_r,y_r))\le p^\star x_1^2\widehat\varphi_1(\xi_1)+\kappa_1^2(|z|)$$

The existence of such a function $\widehat\varphi_1$ is guaranteed because of (A1). Introducing the following compact notation

$$\begin{aligned}\vartheta_1 &= -x_1\nu_1(x_1)-\varphi_1(\xi_1)\widehat\theta-\widehat p x_1\widehat\varphi_1 \qquad (7)\\ \tau_1 &= \Gamma x_1\varphi_1 \qquad (8)\\ \varpi_1 &= \lambda x_1^2\widehat\varphi_1 \qquad (9)\end{aligned}$$

with ν_1 a positive design function to be chosen, it follows from (6) that

$$\begin{aligned}\dot V_1 &\le -x_1^2\nu_1(x_1)+x_1(u-\vartheta_1)+\kappa_1^2(|z|) \qquad (10)\\ &\quad +(\widehat\theta-\theta^\star)^T\Gamma^{-1}(\dot{\widehat\theta}-\tau_1)+\lambda^{-1}(\widehat p-p^\star)(\dot{\widehat p}-\varpi_1)\end{aligned}$$

In the next subsection, it is shown that a similar differential inequality to (10) can be obtained also for system (1) with relative degree $n>1$.

Remark 1 It is worth noting that (7) reduces to a *linear PI control law* when θ and φ_1 are constant functions and when one takes $\dot{\widehat\theta}=\tau_1$. Indeed, in this case, $\widehat\varphi_1=0$ and ν_1 can be taken as a constant feedback gain. See Corollary 1 for another interesting situation.

3.2 Higher relative-degree: $n>1$

We can establish a similar property to (10) for the higher relative-degree case by means of mathematical induction. More precisely, for any $2\le i\le n$, we have designed intermediate smooth functions $\{\vartheta_j\}_{j=1}^i$, $\{\tau_j\}_{j=1}^i$, $\{\varpi_j\}_{j=1}^i$ and have obtained a function V_i of the following form

$$\begin{aligned}V_i &= V_{i-1}+\frac12 x_i^2 := \sum_{j=1}^i\frac12 x_j^2\\ &\quad +\frac12(\widehat\theta-\theta^\star)^T\Gamma^{-1}(\widehat\theta-\theta^\star)+\frac{1}{2\lambda}(\widehat p-p^\star)^2 \qquad (11)\end{aligned}$$

where $x_i=\xi_i-\vartheta_{i-1}$ (for notational coherence, set $\vartheta_0:=y_r$ and $\xi_{n+1}=u$). Morever, V_i satisfies a differential inequality of the type

$$\begin{aligned}\dot V_i &\le -x_1^2(\nu_1-i+1)-\sum_{j=2}^i\nu_j x_j^2+i\kappa_1^2(|z|)\\ &\quad + x_i(\xi_{i+1}-\vartheta_i) \qquad (12)\\ &\quad +\left((\widehat\theta-\theta^\star)^T\Gamma^{-1}-\sum_{j=1}^{i-1}x_{j+1}\frac{\partial\vartheta_j}{\partial\widehat\theta}\right)(\dot{\widehat\theta}-\tau_i)\\ &\quad +\left(\lambda^{-1}(\widehat p-p^\star)-\sum_{j=1}^{i-1}x_{j+1}\frac{\partial\vartheta_j}{\partial\widehat p}\right)(\dot{\widehat p}-\varpi_i)\end{aligned}$$

where ν_1 is a design function and $\{\nu_j\}_{j=2}^i$ are design parameters.

Therefore, at Step $i=n$, we choose the following adaptive laws and controller to annihilate the last three terms in (12)

$$\dot{\widehat\theta}=\tau_n\ ,\quad \dot{\widehat p}=\varpi_n\ ,\quad u=\vartheta_n\ , \qquad (13)$$

the time derivative of the Lyapunov function candidate

$$V_n=\sum_{j=1}^n\tfrac12 x_j^2+\tfrac12(\widehat\theta-\theta^\star)^T\Gamma^{-1}(\widehat\theta-\theta^\star)+\tfrac{1}{2\lambda}(\widehat p-p^\star)^2 \qquad (14)$$

satisfies

$$\dot V_n\le -x_1^2(\nu_1(x_1)-n+1)-\sum_{j=2}^n\nu_j x_j^2+n\kappa_1^2(|z|) \qquad (15)$$

Stability analysis is accomplished in the next section along with the statement of our main results.

4 Main results

We are in a position to state and prove our first main theorem in robust nonlinear integral control.

Theorem 1 *Under Assumptions (A1), (A2) and (A3), if the following local condition holds*

$$\limsup_{s\to0^+}\frac{\gamma_0(s)}{s^2}<+\infty,\ \limsup_{s\to0^+}\frac{\kappa_1^2(s)}{\alpha_0(s)}<+\infty, \qquad (16)$$

then all solutions of the closed-loop system (1)–(13) are bounded and enjoy the following tracking property

$$\lim_{t\to\infty}|y(t)-y_r| = 0 \qquad (17)$$

Remark 2 A sufficient condition for (16) to hold is that the unforced z-system $\dot{z} = q_0(z, 0)$ is locally exponentially stable at the origin (cf. [7, Theorem 6.1], [5, p. 837]). With this in mind, the (small-gain type) local condition (16) is less restrictive than [8, Assumption 5].

Proof. Consider the following Lyapunov function for the entire closed-loop system (1)–(13)

$$V_c \;=\; V_n(x_1, \ldots, x_n, \widehat{\theta}, \widehat{p}) \;+\; \int_0^{V_0(z)} \rho(s)ds \quad (18)$$

where $\rho : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous nondecreasing function with $\rho(s) > 0$ for every $s \geq 0$. With the help of (15) and (A3), differentiating V_c with respect to time yields

$$\begin{aligned} \dot{V}_c \;\leq\; & -x_1^2(\nu_1(x_1) - n + 1) - \sum_{j=2}^{n} \nu_j x_j^2 \\ & +n\kappa_1^2(|z|) - (1-\epsilon)\rho \circ \underline{\alpha}(|z|)\alpha_0(|z|) \\ & +\rho \circ \bar{\alpha} \circ \alpha_0^{-1} \circ \frac{1}{\epsilon}\gamma_0(|x_1|)\gamma_0(|x_1|) \qquad (19) \end{aligned}$$

where $0 < \epsilon < 1$, $\bar{\alpha}$ and $\underline{\alpha}$ are class-$\mathcal{K}_\infty$ functions such that $\underline{\alpha}(|z|) \leq V_0(z) \leq \bar{\alpha}(|z|), \forall z \in \mathbb{R}^{n_0}$. Thanks to the local condition (16), we can find a desired function ρ and, then, a smooth design function ν_1 such that (see [12, 5] for details)

$$\begin{aligned} & (1-\epsilon)\rho \circ \underline{\alpha}(|z|)\alpha_0(|z|) \;\geq\; 2n\kappa_1^2(|z|) \;, \\ & x_1^2(\nu_1(x_1) - n + 1) \\ & \qquad \geq\; 2\rho \circ \bar{\alpha} \circ \alpha_0^{-1} \circ \frac{1}{\epsilon}\gamma_0(|x_1|)\gamma_0(|x_1|) \;. \end{aligned}$$

Consequently,

$$\begin{aligned} \dot{V}_c \;\;\leq\;\; & -\frac{1}{2}x_1^2(\nu_1(x_1) - n + 1) - \sum_{j=2}^{n} \nu_j x_j^2 \\ & -\frac{1-\epsilon}{2}\rho \circ \underline{\alpha}(|z|)\alpha_0(|z|) \qquad (20) \end{aligned}$$

On the basis of (20), noticing that $x_1 = y - y_r$, a direct application of LaSalle's invariance principle completes the proof of Theorem 1. □□□

In special circumstances, our controller (13) reduces to a linear control law for a class of systems with uncertain nonlinearities. More precisely,

Corollary 1 *Under the conditions of Theorem 1, if there is a known upper-bound for p_1 and p_2 and $\{\varphi_i\}_{i=1}^n$ are constant functions, and if κ_2 is a linear function and κ_1 satisfies*

$$\kappa_1 \circ \underline{\alpha}^{-1} \circ \bar{\alpha} \circ \alpha_0^{-1} \circ \frac{1}{\epsilon}\gamma_0(s) \;\leq\; cs \;, \quad \forall s \geq 0 \;\; (21)$$

for some constants $c > 0$ and $0 < \epsilon < 1$, then the controller (13) reduces to a linear control law. In addition, the controller (13) reduces to a classical PI control law when either the relative degree n is one, or the uncertainty meets the strict matching condition, i.e., $\theta_i \equiv 0$ for all $1 \leq i < n$.

Remark 3 The condition (21) simply means that the input-output gain of the ISS z-system (2) with input x_1 and output $\kappa_1(|z|)$ is linearly bounded (cf. [11, 6]). Such an example is $\dot{z} = -z^3 + x_1$, $\kappa_1(|z|) = |z|^3$.

Proof. Since p_1 and p_2 in (A1) are bounded by a known constant, there is no need to introduce an estimate $\widehat{p}$ for $p^\star$. Hence, $\{\varpi_i\}_{i=1}^n$ are void. A careful examination of the control design procedure in Section 3.2 yields that each $\bar{\varphi}_{i+1}$ is constant, and both τ_i and ϑ_i are linear functions if ν_1 can be chosen to be constant. The latter is possible because of (21). The last statement of Corollary 1 is more or less clear. □□□

5 Output feedback

Up to now, we have considered the case when partial ξ-state information is available to the designer. In this section, we study a more practical and challenging situation where only the output y is assumed to be directly measurable. Certainly, the price to be paid for this extension is to study a narrower class of systems than (1), described by

$$\begin{aligned} \dot{\eta} \;&=\; q(\eta, y) \\ \dot{\xi} \;&=\; A\xi \;+\; bu \;+\; \Phi(y)\theta(\eta, y) \qquad (22) \\ y \;&=\; \xi_1 \end{aligned}$$

where A is the $n \times n$ matrix with ones on the upper diagonal and zeros elsewhere, $b = (0, \ldots, 0, 1)^T$ and $\Phi(y) = (\phi_1(y), \ldots, \phi_n(y))^T$. Notice that the major difference between (22) and (1) is that the functions $\{\phi_i\}_{i=1}^n$ in (22) are restricted to depend only on the output y, as in [9, 10]. We assume that the η-system and the uncertain nonlinear function θ satisfy the hypotheses (A1) to (A3). Since

$\{\xi_i\}_{i=2}^n$ are unmeasured, we need to generate some approximate estimates for these states through a "filter" or an "observer". Such filters are formulated as follows:

$$\dot{\zeta} \;=\; (A-Lc)\zeta \;+\; Ly \;+\; bu \qquad (23)$$
$$\dot{\Xi} \;=\; (A-Lc)\Xi \;+\; \Phi(y) \qquad (24)$$

where $c = (1,0,\ldots,0)$ and $L = (L_1,\ldots,L_n)^T$ is chosen to make the matrix $A - Lc$ Hurwitz. Let $\theta^\star = \theta(\eta_r, y_r)$ and $\widetilde{\xi} = \xi - \zeta - \Xi\theta^\star$. Direct computation yields

$$\dot{\widetilde{\xi}} \;=\; (A-Lc)\widetilde{\xi} \;+\; \Phi(y)\,(\theta(\eta,y) - \theta(\eta_r,y_r)) \qquad (25)$$

Noting that $\eta = z + \eta_r$ and $y = x_1 + y_r$, with Assumption (A1), it is easy to show that system (25) is ISS when (z, x_1) is considered as the input. As a result, the cascade system comprised of the ISS z-system (2) with input x_1 and the ISS $\widetilde{\xi}$-system (25) with input (z, x_1) is again ISS with respect to x_1 and, more precisely, fulfills (A3). See [11, 6]. The integral control problem, stated in Section 2, is solvable for system (22) with output measurements if we apply the design procedure in Section 3 to the following augmented system

$$\begin{aligned}
&\dot{z} = q_0(z, x_1)\\
&\dot{\widetilde{\xi}} = (A-Lc)\widetilde{\xi} + \Phi(y)\,(\theta(\eta,y) - \theta(\eta_r,y_r))\\
&\dot{\xi}_1 = \zeta_2 + (\Xi_{(2)} + \phi_1)\theta^\star + \widetilde{\xi}_2\\
&\qquad +\phi_1(y)\,(\theta(\eta,y) - \theta(\eta_r,y_r))\\
&\dot{\zeta}_2 = \zeta_3 + L_2(y - \zeta_1)\\
&\vdots\\
&\dot{\zeta}_n = u + L_n(y-\zeta_1)\\
&x_1 = \xi_1 - y_r
\end{aligned} \qquad (26)$$

with $\Xi_{(2)}$ standing for the second row vector of the matrix Ξ. Notice that the partial $(\xi_1, \zeta_2, \ldots, \zeta_n)$-state of (26) is accessible for feedback design. We refer the reader to [5] for details about how to bring output-feedback control back to a *partial-state* feedback control problem. Our second main theorem in robust nonlinear integral control is stated below, whose proof follows the similar lines in the proof of Theorem 1 and is omitted for want of space.

Theorem 2 *Under Assumptions (A1) to (A3), if (16) holds, then the Problem of Robust Integral Control is solvable for system (22) using dynamic, output-feedback* $\dot{\chi} = \nu(\chi, y)$, $u = \mu(\chi, y)$.

In the sequel, we use a practical example of fan speed control to illustrate our proposed output-feedback integral control method. This system was studied by Freeman and Kokotović [2, p. 223] via full-state feedback.

Example 1 *(Fan speed control)* The dynamics of a fan driven by a DC motor is described by

$$\begin{aligned}
J_1\dot{v} &= k_1 I \;-\; \tau_L \;-\; \tau_D(v)\\
J_2\dot{I} &= u_0 \;-\; k_2 v \;-\; RI\\
y &= v
\end{aligned} \qquad (27)$$

where v is the fan speed, I is the armature current and u_0 is the armature voltage which is considered as the input. Although unnecessary in our framework, we take the simplifying assumption, used in [2, p. 223], that the positive constants J_1, J_2, k_1, k_2 and R are known. But, we assume that τ_L is an unknown constant load torque and the nonlinearity $\tau_D(v)$ is an uncertain drag torque. It is further assumed that $\tau_D(v)$ verifies (A3), i.e., for each v_r, there are an (unknown) constant $p > 0$ and a (known) smooth function κ such that

$$|\tau_D(v) - \tau_D(v_r)| \;\le\; p\kappa(|v - v_r|),\ \forall\, v \in \mathbb{R} \qquad (28)$$

The control problem is the regulation of the fan speed v to a desired value v_r, irrespective of unknown τ_L, $\tau_D(v)$ and unmeasured I. Introducing the change of coordinates $\xi_1 = v$, $\xi_2 = k_1 I/J_1$ and feedback $u = k_1(u_0 - k_2 v - RI)/(J_1 J_2)$, the model (27) is brought into a system (22) without dynamic uncertainty, i.e.,

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 \;+\; \theta(y)\\
\dot{\xi}_2 &= u\\
y &= \xi_1
\end{aligned} \qquad (29)$$

where $\theta(y) = -(\tau_L + \tau_D(y))/J_1$. Let $\theta^\star = \theta(y_r)$ with $y_r = v_r$ the desired speed.

Since system (29) satisfies the conditions of Theorem 2, our output-feedback method is applicable to obtain a desired nonlinear integral controller. As a side remark, reduced-order filters, instead of full-order filters (23)-(24), can be adopted. This is demonstrated as follows.

For any $L > 0$, introduce the reduced-order filters

$$\begin{aligned}
\dot{\zeta} &= -L\zeta + -L^2 y + u\\
\dot{\Xi} &= -L\Xi - L
\end{aligned} \qquad (30)$$

Let $z = \xi_2 - L\xi_1 - \zeta - \Xi\theta^\star$. We obtain the following transformed system for feedback design

$$\begin{aligned} \dot{z} &= -Lz - L(\theta(y) - \theta(y_r)) \\ \dot{\xi}_1 &= \zeta + L\xi_1 + (\Xi+1)\theta^\star + z \\ &\quad + (\theta(y) - \theta(y_r)) \\ \dot{\zeta} &= u - L\zeta - L^2 y \end{aligned} \tag{31}$$

Clearly, the z-system is ISS with input $x_1 = y - y_r$ and has the quadratic function $V_0 = \frac{1}{2}z^2$ as an ISS-Lyapunov function. A direct application of the control design procedure in Section 3 generates a desired dynamic output-feedback controller

$$\begin{aligned} \dot{\widehat{\theta}} &= \Gamma x_1(\Xi+1) - \Gamma x_2 \tfrac{\partial\vartheta_1}{\partial\xi_1}(\Xi+1) \\ \dot{\widehat{p}} &= \tfrac{\lambda}{J_1^2} x_1^2 \widehat{\kappa}(x_1)(J_1 + L\widehat{\kappa}(x_1)) \\ &\quad + \frac{\lambda}{4J_1^2} x_2^2 \widehat{\kappa}^2(x_1)(\frac{\partial\vartheta_1}{\partial\xi_1})^2 \\ u &= -\nu_2 x_2 - x_1 + L\zeta + L^2 y + \frac{\partial\vartheta_1}{\partial\xi_1}(\zeta + L\xi_1) \\ &\quad + \frac{\partial\vartheta_1}{\partial\xi_1}(\Xi+1)\widehat{\theta} - \left(\frac{\widehat{p}\widehat{\kappa}^2}{4J_1^2} + \frac{1}{L}\right)(\frac{\partial\vartheta_1}{\partial\xi_1})^2 x_2 \\ &\quad - \frac{\partial\vartheta_1}{\partial\Xi}(L\Xi + L) + \frac{\partial\vartheta_1}{\partial\widehat{\theta}}\dot{\widehat{\theta}} + \frac{\partial\vartheta_1}{\partial\widehat{p}}\dot{\widehat{p}} \end{aligned}$$

where $\widehat{\kappa}(x_1)$ is any smooth function dominating $|\int_0^1 \kappa'(\lambda|x_1|)d\lambda|$ for each $x_1 \in \mathbb{R}$, and $\vartheta_1(\xi_1, \Xi, \widehat{\theta}, \widehat{p}) = -\nu_1 x_1 - L_1\xi_1 - L^{-1}x_1 - (\Xi+1)\widehat{\theta} - \widehat{p}x_1\widehat{\kappa}J_1^{-2}(J_1 + L\widehat{\kappa})$ for some constants $\nu_1, \nu_2 > 0$.

When a bound for p is known, a simpler output-feedback controller can be obtained.

6 Concluding remarks

In this paper, we have considered the problem of robust nonlinear integral control for a class of systems with dynamic uncertainties. Both partial-state and output feedback cases are addressed. Our controllers reduce to linear control laws if the nominal system is linear and driven by the dynamic uncertainty whose input-output gain is linearly bounded. The linear controller becomes a traditional PI controller if, additionally, the relative degree is one, or the uncertainty falls into the input space (i.e., strict matching condition). (see Remark 1). A constructive controller design procedure is presented and illustrated in a practical example of fan speed control.

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